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FENSOR ENERGÍA-MOMENTO $T^{\mu\nu} = \dfrac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial^\nu \phi - g^{\mu\nu} \mathcal{L}$

a) Comprobar que es simétrico $T^{\alpha\beta} = T^{\beta\alpha}$

b) Calcular sus componentes.

a) Si $T^{\alpha\beta} = T^{\beta\alpha}$ debe ser

$$\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi)} \partial^\beta \phi - g^{\alpha\beta} \mathcal{L} = \frac{\partial \mathcal{L}}{\partial(\partial_\beta \phi)} \partial^\alpha \phi - g^{\beta\alpha} \mathcal{L}$$

pero sabemos que $g^{\alpha\beta} = g^{\beta\alpha}$, entonces si los primeros términos son iguales

$$\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi)} \partial^\beta \phi = \frac{\partial \mathcal{L}}{\partial(\partial_\beta \phi)} \partial^\alpha \phi \Rightarrow T^{\alpha\beta} = T^{\beta\alpha}$$

$$\boxed{\mathcal{L} = \frac{1}{2} \partial_\alpha \phi \, \partial^\alpha \phi - U(\phi)}$$

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi)} &= \frac{1}{2} \partial^\alpha \phi \qquad \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi)} \cdot \partial^\beta \phi = \frac{1}{2} \partial^\alpha \phi \, \partial^\beta \phi \\ \frac{\partial \mathcal{L}}{\partial(\partial_\beta \phi)} &= \frac{1}{2} \partial^\beta \phi \qquad \frac{\partial \mathcal{L}}{\partial(\partial_\beta \phi)} \partial^\alpha \phi = \frac{1}{2} \partial^\beta \phi \, \partial^\alpha \phi \end{aligned}\right\} \text{SON IGUALES}$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi)} \partial^\beta \phi = \frac{\partial \mathcal{L}}{\partial(\partial_\beta \phi)} \partial^\alpha \phi \quad \text{y} \quad g^{\alpha\beta} \mathcal{L} = g^{\beta\alpha} \mathcal{L} \Rightarrow T^{\alpha\beta} = T^{\beta\alpha}$$

b) CALCULAR LOS COMPONENTES DEL TENSOR

Javier calculó $\longrightarrow$ $T^{00} = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} (\vec{\nabla} \phi)^2 + U(\phi)$

$$T^{0i} = -\dot{\phi} \, \partial_i \phi$$

• Para $i \neq j$ $\quad T^{ij} = \dfrac{\partial \mathcal{L}}{\partial(\partial_i \phi)} \partial^j \phi - \underbrace{g^{ij}}_{= 0} \mathcal{L} = \dfrac{\partial \mathcal{L}}{\partial(\partial_i \phi)} \partial^j \phi$

NOTA

HOJA Nº 2

FECHA

$$T^{ij} = -\frac{1}{2}\,\partial^i\phi\,\partial^j\phi \quad \text{con } \partial^i = -\partial_i$$

$$\boxed{T^{ij} = -\frac{1}{2}\,\partial_i\phi\,\partial_j\phi}$$ ya hemos demostrado que $T^{ij} = T^{ji}$

- Para $i = j$

$$T^{ii} = \frac{\partial\mathcal{L}}{\partial(\partial_i\phi)}\,\partial^i\phi - \underbrace{g^{ii}}_{(-1)}\mathcal{L} = \frac{\partial\mathcal{L}}{\partial(\partial_i\phi)}\,\partial^i\phi + \mathcal{L}$$

$$T^{ii} = -\frac{1}{2}\,\partial_i\phi\,\partial^i\phi + \mathcal{L} = \frac{1}{2}(\partial_i\phi)^2 + \mathcal{L}$$

$$= \frac{1}{2}(\partial_i\phi)^2 + \frac{1}{2}\dot{\phi}^2 - \sum_{k=1}^{3}\frac{1}{2}(\partial_k\phi)^2 - U(\phi)$$

se anula este término

$$\frac{1}{2}(\partial_i\phi)^2 - \sum_{k=1}^{3}\frac{1}{2}(\partial_k\phi)^2 = -\frac{1}{2}\left(1-\delta_i^j\right)(\partial_j\phi)^2$$

$$\boxed{T^{ii} = \frac{1}{2}(\partial_i\phi)^2 - \frac{1}{2}\left(1-\delta_i^j\right)(\partial_j\phi)^2 - U(\phi)}$$

NOTA